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ESERCIZI SVOLTI
LOGARITMI

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Determinare direttamente il valore dei seguenti logaritmi:

1) $\log_{11} 1 = x$ $\quad 11^x = 1 \quad 11^x = 11^0 \qquad x = 0$.

2) $\log_6 6 = x$ $\quad 6^x = 6 \qquad 6^x = 6^1 \quad x = 1$.

3) $\log_7 49 = x$ $\quad 7^x = 49 \quad 7^x = 7^2 \qquad x = 2$.

4) $\log_3 27 = x$ $\quad 3^x = 27 \quad 3^x = 3^3 \qquad x = 3$.

5) $\log_2 16 = x$ $\quad 2^x = 16 \quad 2^x = 2^4 \qquad x = 4$.

6) $\log_2 32 = x$ $\quad 2^x = 32 \quad 2^x = 2^5 \qquad x = 5$.

7) $\log_8 2 = x$ $\quad 8^x = 2 \qquad 2^{3x} = 2 \quad 3x = 1 \quad x = \frac{1}{3}$.

8) $\log_6 36 = x$ $\quad 6^x = 36 \quad 6^x = 6^2 \qquad x = 2$.

9) $\log_{36} 6 = x$ $\quad 36^x = 6 \quad 6^{2x} = 6 \qquad 2x = 1 \quad x = \frac{1}{2}$.

10) $\log_9 81 = x$ $\quad 9^x = 81 \quad 3^{2x} = 3^4 \qquad 2x = 4 \quad x = \frac{4}{2} \quad x = 2$.

11) $\log_9 27 = x$ $\quad 9^x = 27 \quad 3^{2x} = 3^3 \qquad 2x = 3 \quad x = \frac{3}{2}$.

12) $\log_{\frac{1}{3}} 3 = x$ $\quad \left(\frac{1}{3}\right)^x = 3 \qquad \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-1} \qquad x = -1$.

13) $\log_{\frac{1}{5}} 25 = x$ $\quad \left(\frac{1}{5}\right)^x = 25 \qquad \left(\frac{1}{5}\right)^x = 5^2 \qquad \left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-2} \qquad x = -2$.

14) $\log_{\frac{1}{5}} 125 = x$ $\quad \left(\frac{1}{5}\right)^x = 125 \qquad \left(\frac{1}{5}\right)^x = 5^3 \qquad \left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-3} \qquad x = -3$.

15) $\log_{\frac{1}{3}} 27 = x$ $\quad \left(\frac{1}{3}\right)^x = 27 \qquad \left(\frac{1}{3}\right)^x = 3^3 \qquad \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-3} \qquad x = -3$.

16) $\log_{\frac{1}{2}} 64 = x$ $\quad \left(\frac{1}{2}\right)^x = 64 \qquad \left(\frac{1}{2}\right)^x = 2^6 \qquad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-6} \qquad x = -6$.

17) $\log_{\frac{1}{2}} \frac{1}{2} = x$ $\quad \left(\frac{1}{2}\right)^x = \frac{1}{2} \qquad \left(\frac{1}{2}\right)^x = \frac{1}{2^1} \qquad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{1} \qquad x = 1$.

18) $\log_{\frac{1}{2}} \frac{1}{4} = x$ $\quad \left(\frac{1}{2}\right)^x = \frac{1}{4} \qquad \left(\frac{1}{2}\right)^x = \frac{1}{2^2} \qquad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{2} \qquad x = 2$.

19) $\log_{\frac{1}{7}} \frac{1}{49} = x$ $\quad \left(\frac{1}{7}\right)^x = \frac{1}{49} \qquad \left(\frac{1}{7}\right)^x = \frac{1}{7^2} \qquad \left(\frac{1}{7}\right)^x = \left(\frac{1}{7}\right)^{2} \qquad x = 2$.

20) $\log_2 \frac{1}{64} = x$ $\quad 2^x = \frac{1}{64} \qquad 2^x = \left(\frac{1}{2}\right)^6 \qquad 2^x = 2^{-6} \qquad x = -6$

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21) $\log_2 \frac{1}{128} = x$ $\quad 2^x = \frac{1}{128} \quad 2^x = \left(\frac{1}{2}\right)^7 \quad 2^x = 2^{-7} \quad x = -7$

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22) $\log_{\frac{1}{2}} \frac{1}{128} = x \quad \left(\frac{1}{2}\right)^x = \frac{1}{128} \quad \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^7 \quad x = 7$.

23) $\log_{\frac{2}{3}} \frac{4}{9} = x \quad \left(\frac{2}{3}\right)^x = \frac{4}{9} \quad \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2 \quad x = 2$.

24) $\log_{\frac{2}{3}} \frac{16}{81} = x \quad \left(\frac{2}{3}\right)^x = \frac{16}{81} \quad \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^4 \quad x = 4$.

25) $\log_{\frac{2}{5}} \frac{25}{4} = x \quad \left(\frac{2}{5}\right)^x = \frac{25}{4} \quad \left(\frac{2}{5}\right)^x = \left(\frac{5}{2}\right)^2 \quad \left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^{-2} \quad x = -2$

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26) $\log_{\frac{3}{5}} \frac{125}{27} = x \quad \left(\frac{3}{5}\right)^x = \frac{125}{27} \quad \left(\frac{3}{5}\right)^x = \left(\frac{5}{3}\right)^3 \quad \left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^{-3} \quad x = -3$

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27) $\log_{\frac{9}{4}} \frac{3}{2} = x \quad \left(\frac{9}{4}\right)^x = \frac{3}{2} \quad \left(\frac{3}{2}\right)^{2x} = \frac{3}{2} \quad 2x = 1 \quad x = \frac{1}{2}$.

28) $\log_{\frac{49}{36}} \frac{6}{7} = x \quad \left(\frac{49}{36}\right)^x = \frac{6}{7} \quad \left(\frac{7}{6}\right)^{2x} = \left(\frac{7}{6}\right)^{-1} \quad 2x = -1 \quad x = -\frac{1}{2}$.

29) $\log_2 \sqrt{2} = x \quad 2^x = \sqrt{2} \quad 2^x = 2^{\frac{1}{2}} \quad x = \frac{1}{2}$.

30) $\log_2 \sqrt[3]{2} = x \quad 2^x = \sqrt[3]{2} \quad 2^x = 2^{\frac{1}{3}} \quad x = \frac{1}{3}$.

31) $\log_2 \sqrt[3]{4} = x \quad 2^x = \sqrt[3]{4} \quad 2^x = \sqrt[3]{2^2} \quad 2^x = 2^{\frac{2}{3}} \quad x = \frac{2}{3}$.

32) $\log_3 \sqrt[3]{3} = x \quad 3^x = \sqrt[3]{3} \quad 3^x = 3^{\frac{1}{3}} \quad x = \frac{1}{3}$.

33) $\log_3 \sqrt[3]{9} = x \quad 3^x = \sqrt[3]{9} \quad 3^x = \sqrt[3]{3^2} \quad 3^x = 3^{\frac{2}{3}} \quad x = \frac{2}{3}$.

34) $\log_{\frac{1}{3}} \sqrt[3]{9} = x \quad \left(\frac{1}{3}\right)^x = \sqrt[3]{9} \quad \left(\frac{1}{3}\right)^x = 3^{\frac{2}{3}} \quad \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-\frac{2}{3}} \quad x = -\frac{2}{3}$.

35) $\log_{\frac{1}{5}} \sqrt[3]{5} = x \quad \left(\frac{1}{5}\right)^x = \sqrt[3]{5} \quad \left(\frac{1}{5}\right)^x = 5^{\frac{1}{3}} \quad \left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{-\frac{1}{3}} \quad x = -\frac{1}{3}$.

Dato il logaritmo e la base determinare il numero:

1) $\log_3 x = 0 \quad 3^0 = x \quad x = 3^0 \quad x = 1$.
2) $\log_3 x = 1 \quad 3^1 = x \quad x = 3^1 \quad x = 3$.
3) $\log_3 x = 2 \quad 3^2 = x \quad x = 3^2 \quad x = 9$.
4) $\log_3 x = 0 \quad 3^0 = x \quad x = 3^0 \quad x = 1$.
5) $\log_2 x = 4 \quad 2^4 = x \quad x = 2^4 \quad x = 16$.
6) $\log_8 x = 2 \quad 8^2 = x \quad x = 8^2 \quad x = 64$.

7) $\log_{10} x = 2$ $\quad 10^2 = x \quad x = 10^2 \quad x = 100$.

8) $\log_{10} x = 4$ $\quad 10^4 = x \quad x = 10^4 \quad x = 10000$.

9) $\log_2 x = \frac{1}{2}$ $\quad 2^{\frac{1}{2}} = x \quad x = 2^{\frac{1}{2}} \quad x = \sqrt{2}$.

10) $\log_4 x = \frac{1}{2}$ $\quad 4^{\frac{1}{2}} = x \quad x = 4^{\frac{1}{2}} \quad x = \sqrt{4} \quad x = 2$.

11) $\log_{27} x = \frac{1}{3}$ $\quad 27^{\frac{1}{3}} = x \quad x = 27^{\frac{1}{3}} \quad x = \sqrt[3]{27} \quad x = 3$.

12) $\log_{\frac{1}{4}} x = -1$ $\quad \left(\frac{1}{4}\right)^{-1} = x \quad x = \left(\frac{1}{4}\right)^{-1} \quad x = 4$.

13) $\log_4 x = -2$ $\quad 4^{-2} = x \quad x = 4^{-2} \quad x = \left(\frac{1}{4}\right)^2 \quad x = \frac{1}{16}$.

14) $\log_{11} x = -2$ $\quad 11^{-2} = x \quad x = 11^{-2} \quad x = \left(\frac{1}{11}\right)^2 \quad x = \frac{1}{121}$.

15) $\log_{\frac{1}{5}} x = 3$ $\quad \left(\frac{1}{5}\right)^3 = x \quad x = \left(\frac{1}{5}\right)^3 \quad x = \frac{1}{125}$.

16) $\log_{\frac{1}{4}} x = -3$ $\quad \left(\frac{1}{4}\right)^{-3} = x \quad x = \left(\frac{1}{4}\right)^{-3} \quad x = 4^3 \quad x = 64$.

17) $\log_{\frac{1}{2}} x = -6$ $\quad \left(\frac{1}{2}\right)^{-6} = x \quad x = \left(\frac{1}{2}\right)^{-6} \quad x = 2^6 \quad x = 64$.

18) $\log_{0,1} x = 2$ $\quad \log_{\frac{1}{10}} x = 2 \quad \left(\frac{1}{10}\right)^2 = x \quad x = \left(\frac{1}{10}\right)^2 \quad x = \frac{1}{100}$.

19) $\log_{0,2} x = 2$ $\quad \log_{\frac{1}{5}} x = 2 \quad \left(\frac{1}{5}\right)^2 = x \quad x = \left(\frac{1}{5}\right)^2 \quad x = \frac{1}{25}$.

20) $\log_{0,5} x = 3$ $\quad \log_{\frac{1}{2}} x = 3 \quad \left(\frac{1}{2}\right)^3 = x \quad x = \left(\frac{1}{2}\right)^3 \quad x = \frac{1}{8}$.

Determinare la base dei seguenti logaritmi:

1) $\log_x 4 = 2$ $\quad x^2 = 4 \quad x^2 = 2^2 \quad x = 2$.

2) $\log_x 9 = 2$ $\quad x^2 = 9 \quad x^2 = 3^2 \quad x = 3$.

3) $\log_x 144 = 2$ $\quad x^2 = 144 \quad x^2 = 12^2 \quad x = 12$.

4) $\log_x 27 = 3$ $\quad x^3 = 27 \quad x^3 = 3^3 \quad x = 3$.

5) $\log_x 8 = 3$ $\quad x^3 = 8 \quad x^3 = 2^3 \quad x = 2$.

6) $\log_x 1000 = 3$ $\quad x^3 = 1000 \quad x^3 = 10^3 \quad x = 10$.

7) $\log_x 10000 = 4$ $\quad x^4 = 10000 \quad x^4 = 10^4 \quad x = 10$.

8) $\log_x 32 = 5$ $\quad x^5 = 32 \quad x^5 = 2^5 \quad x = 2$.

9) $\log_x 243 = 5$ $\quad x^5 = 243 \quad x^5 = 3^5 \quad x = 3$.

10) $\log_x 64 = 3$ $\quad x^3 = 64 \quad x^3 = 4^3 \quad x = 4$.

11) $\log_x 125 = 3$ $\quad x^3 = 125 \quad x^3 = 5^3 \quad x = 5$.

12) $\log_x 64 = 6$ $\quad x^6 = 64 \quad x^6 = 2^6 \quad x = 2$.

13) $\log_x 169 = 2$ $\quad x^2 = 169 \quad x^2 = 13^2 \quad x = 13$.

14) $\log_x 2 = -1$ $\quad x^{-1} = 2 \quad x^{-1} = 2^1 \quad x^{-1} = \left(\frac{1}{2}\right)^{-1} \quad x = \frac{1}{2}$.

15) $\log_x 16 = -4 \quad x^{-4} = 16 \quad x^{-4} = 2^4 \qquad x^{-4} = \left(\frac{1}{2}\right)^{-4} \qquad x = \frac{1}{2}$.

16) $\log_x 8 = -3 \quad x^{-3} = 8 \quad x^{-3} = 2^3 \qquad x^{-3} = \left(\frac{1}{2}\right)^{-3} \qquad x = \frac{1}{2}$.

17) $\log_x 25 = -2 \quad x^{-2} = 25 \quad x^{-2} = 5^2 \qquad x^{-2} = \left(\frac{1}{5}\right)^{-2} \qquad x = \frac{1}{5}$.

18) $\log_x \sqrt{2} = \frac{1}{2} \quad x^{\frac{1}{2}} = \sqrt{2} \quad x^{\frac{1}{2}} = 2^{\frac{1}{2}} \qquad x = 2$.

19) $\log_x \sqrt{3} = \frac{1}{2} \quad x^{\frac{1}{2}} = \sqrt{3} \quad x^{\frac{1}{2}} = 3^{\frac{1}{2}} \qquad x = 3$.

20) $\log_x \sqrt[3]{2} = \frac{1}{3} \quad x^{\frac{1}{3}} = \sqrt[3]{2} \quad x^{\frac{1}{3}} = 2^{\frac{1}{3}} \qquad x = 2$.

21) $\log_x \sqrt[5]{2^2} = \frac{2}{5} \quad x^{\frac{2}{5}} = \sqrt[5]{2^2} \qquad x^{\frac{2}{5}} = 2^{\frac{2}{5}} \quad x = 2$.

22) $\log_x \sqrt[3]{\frac{1}{9}} = \frac{2}{3} \quad x^{\frac{2}{3}} = \sqrt[3]{\frac{1}{9}} \quad x^{\frac{2}{3}} = \left(\frac{1}{9}\right)^{\frac{1}{3}} \qquad x^{\frac{2}{3}} = \left(\frac{1}{3}\right)^{\frac{2}{3}} \qquad x = \frac{1}{3}$.

23) $\log_x \sqrt[3]{\frac{1}{25}} = \frac{2}{3} \quad x^{\frac{2}{3}} = \sqrt[3]{\frac{1}{25}} \qquad x^{\frac{2}{3}} = \left(\frac{1}{25}\right)^{\frac{1}{3}} \qquad x^{\frac{2}{3}} = \left(\frac{1}{5}\right)^{\frac{2}{3}} \qquad x = \frac{1}{5}$.

Applicando le proprietà sui logaritmi trasformare i seguenti logaritmi neperiani in somme algebriche di logaritmi:

1) $\log 2a^2b^3 =$

$= \log 2 + \log a^2 + \log b^3 = \log 2 + 2\log a + 3\log b$.

2) $\log 3a^4\sqrt{b} = = \log 3 + \log a^4 + \log\sqrt{b} = \log 3 + 4\log a + \log b^{\frac{1}{2}} = \log 3 + 4\log a + \frac{1}{2}\log b$.

3) $\log\frac{5x^2}{x+3} =$

$= \log 5x^2 - \log(x+3) = \log 5 + 2\log x - \log(x+3)$.

4) $\log\frac{27x^3y^2}{\sqrt[3]{y}} =$

$= \log 27x^3y^2 - \log\sqrt[3]{y} = \log 3^3x^3y^2 - \log y^{\frac{1}{3}} =$

$= 3\log 3 + 3\log x + 2\log y - \frac{1}{3}\log y = 3(\log 3 + \log x) + \frac{5}{3}\log y$.

5) $\log\frac{81x^4y}{\sqrt[4]{z}} =$

$= \log 81x^4y - \log\sqrt[4]{z} = \log 3^4x^4y - \log z^{\frac{1}{4}} =$

$= 4\log 3 + 4\log x + \log y - \frac{1}{4}\log z = 4(\log 3 + \log x) - \frac{1}{4}\log z$.

6) $\log\frac{5a^4b}{c^2d^3} =$

$$= \log 5a^4b - \log c^2d^3 = \log 5 + \log a^4 + \log b - \log c^2 - \log d^3 =$$
$$= \log 5 + 4\log a + \log b - 2\log c - 3\log d \;.$$

7) $\log\frac{49a^5b^3}{cd^4} =$

$$= \log 49a^5b^3 - \log cd^4 = \log 7^2 + \log a^5 + \log b^3 - \log c - \log d^4 =$$
$$= 2\log 7 + 5\log a + 3\log b - \log c - 4\log d \;.$$

Tenendo presente le proprietà sui logaritmi ridurre ad un unico logaritmo neperiano ciascuna delle seguenti espressioni:

1) $\log 5 + \log 9 - \frac{1}{2}\log 25 + \log 27 =$

$$= \log 5 + \log 3^2 - \frac{1}{2}\log 5^2 + \log 3^3 = \log 5 + 2\log 3 - \frac{2}{2}\log 5 + 3\log 3 =$$
$$= \log 5 + 2\log 3 - \log 5 + 3\log 3 = 5\log 3 \;.$$

2) $\log 12 - \log 6 + \log 125 - \frac{1}{5}\log 32 - \log 25 =$

$$= \log(3\cdot 2^2) - \log(2\cdot 3) + \log 5^3 - \frac{1}{5}\log 2^5 - \log 5^2 =$$
$$= \log 3 + \log 2^2 - (\log 2 + \log 3) + \log 5^3 - \frac{1}{5}\log 2^5 - \log 5^2 =$$
$$= \log 3 + 2\log 2 - \log 2 - \log 3 + 3\log 5 - \frac{5}{5}\log 2 - 2\log 5 =$$
$$= \log 3 + 2\log 2 - \log 2 - \log 3 + 3\log 5 - \log 2 - 2\log 5 = \log 5 \;.$$

3) $2\log 2 - \frac{1}{3}\log 8 + 2\log 3 - \log\frac{3}{5} - 3\log 2 - \frac{1}{2}\log 25 =$

$$= 2\log 2 - \frac{1}{3}\log 2^3 + 2\log 3 - (\log 3 - \log 5) - 3\log 2 - \frac{1}{2}\log 5^2 =$$
$$= 2\log 2 - \log 2 + 2\log 3 - \log 3 + \log 5 - 3\log 2 - \log 5 =$$
$$= \log 3 - 2\log 2 =$$
$$= \log 3 - \log 2^2 = \log 3 - \log 4 = \log\frac{3}{4} \;.$$

4) $\log 9 - \frac{1}{6}\log 64 - \log 12 + \log\frac{8}{3} - \frac{1}{4}\log 81 + \log 18 =$

$$= \log 3^2 - \frac{1}{6}\log 2^6 - \log(2^2\cdot 3) + \log\frac{2^3}{3} - \frac{1}{4}\log 3^4 + \log(2\cdot 3^2) =$$
$$= 2\log 3 - \log 2 - (\log 2^2 + \log 3) + \log 2^3 - \log 3 - \log 3 + \log 2 + \log 3^2 =$$
$$= 2\log 3 - \log 2 - (2\log 2 + \log 3) + 3\log 2 - \log 3 - \log 3 + \log 2 + 2\log 3 =$$
$$= 2\log 3 - \log 2 - 2\log 2 - \log 3 + 3\log 2 - \log 3 - \log 3 + \log 2 + 2\log 3 =$$
$$= \log 2 + \log 3 =$$
$$= \log(2\cdot 3) = \log 6 \;.$$

Applicando le proprietà calcolare i seguenti logaritmi :

1) $\log_3 \frac{27\sqrt{3}}{\sqrt[3]{3}} =$

$= \log_3 27 + \log_3 \sqrt{3} - \log_3 \sqrt[3]{3} =$

$= \log_3 3^3 + \log_3 3^{\frac{1}{2}} - \log_3 3^{\frac{1}{3}} =$

$= 3\log_3 3 + \frac{1}{2}\log_3 3 - \frac{1}{3}\log_3 3 =$

osservando che $\log_3 3 = 1$ si ha:

$= 3 + \frac{1}{2} - \frac{1}{3} =$

$= \frac{18 + 3 - 2}{6} =$

$= \frac{19}{6}$.

2) $\log_2 \frac{16\sqrt{2}}{\sqrt[4]{2}} =$

$= \log_2 16 + \log_2 \sqrt{2} - \log_2 \sqrt[4]{2} =$

$= \log_2 2^4 + \log_2 2^{\frac{1}{2}} - \log_2 2^{\frac{1}{4}} =$

$= 4\log_2 2 + \frac{1}{2}\log_2 2 - \frac{1}{4}\log_2 2 =$

osservando che $\log_2 2 = 1$ si ha:

$= 4 + \frac{1}{2} - \frac{1}{4} = \frac{16 + 2 - 1}{4} = \frac{17}{4}$.

Torna su